IT'S A PUZZLEMENT By JON EVANS

Deserts of Prime Numbers

Define a *desert* of prime numbers as a sequence of k consecutive integers $\{n,\ldots,n+k-1\}$ none of which is a prime number. How big can such a desert be? For any k that allows a k-sized desert, can you specify a starting integer n for a desert of size k? Furthermore, for any k, what is the maximum number of non-overlapping k-sized deserts? Can you also specify the starting integers for a set of this many non-overlapping k-sized deserts? Even if you cannot answer all these questions for any k, can you answer them for $k=10^{100}$ specifically?

Ping-Pong Team Strategy

Two teams of ping-pong players, Teams A and B, face off in matches of one on one. Each match ends when a player scores one point and the losing player is eliminated from further play. Individual player strength, S, is the average number of seconds until a player gives up a point. The first team to run out of players loses.

Before each match, Team B first selects a player and then Team A selects. What are the best and worst, respectively, possible strategies for Team B and the corresponding probabilities of winning?

Team A		Team B	
Player	Strength (sec.)	Player	Strength (sec.)
A1	40	B1	90
A2	30	B2	20
A3	25	B3	15
A4	20	B4	10
A5	15	B5	5

What about for Team A? What if Team A selects first?

Here is Clive Keatinge's solution.

Because each player's loss rate is independent of the opposing player, the total time that elapses before all of the players on a team have lost is independent of the ordering of opposing players. Because the loss rate is constant for each player, the total time that elapses before all of the players on a team have lost is the sum of five exponential distributions with different rates, which is a hypoexponential distribution with probability density function

$$f(x)=\sum_{i=1}^{5}\lambda_i e^{-\lambda_i x}\prod_{j=1,\,j\neq i}^{5}\left(\frac{\lambda_j}{\lambda_j-\lambda_i}\right)$$

where the λs are the loss rates. The probability that the total time exceeds x is then $1-F(x)=\sum_{i=1}^{5}e^{-\lambda_i x}\prod_{j=1,\,j\neq i}^{5}\left(\frac{\lambda_j}{\lambda_j-\lambda_i}\right)$.

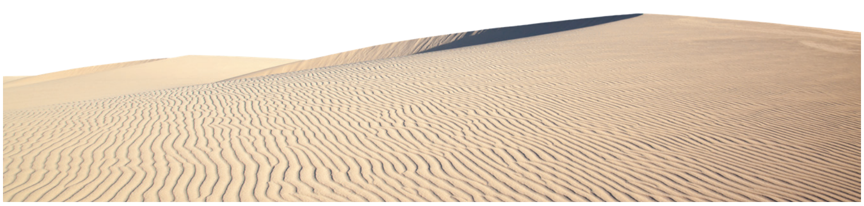
To produce the probability of a win for Team B, multiply the probability density function for Team A by the

probability that the total time for Team B exceeds x, and then integrate over x. If λs represent the loss rates for Team A, and the μs for Team B, then the result is

$$\sum_{i=1}^{5}\sum_{r=1}^{5}\left(\frac{\lambda_i}{\lambda_i-\mu_r}\right)\prod_{j=1,\,j\neq i}^{5}\left(\frac{\lambda_j}{\lambda_j-\lambda_i}\right)\prod_{s=1,\,s\neq r}^{5}\left(\frac{\mu_s}{\mu_s-\mu_r}\right).$$

Plugging in the loss rates given yields a win probability of 0.4909 for Team B, regardless of the ordering of the players on either team.

A solution was also submitted by Bob Conger.



Know the answer? Send your solution to ar@casact.org.