IT'S A PUZZLEMENT By JON EVANS

Turning Into Traffic

You are at a stop sign on a two-lane road that intersects a four-lane (two lanes going each way) road without a stop sign. In each individual lane of the four-lane road, on average, a car passes by the stop sign every four seconds. Given your car's performance, cars on the intersecting road need to be at least four seconds away for you to safely make the turn. If you are in a right-hand traffic country, like the U.S, how long do you expect to wait to make a right-hand turn? How long is it for a left-hand turn? What if you are unlucky and have to wait a long time (worst 10%, worst 1%, …)?

Estimate the volume

The SHA-256 hash: 88b8448136047d588c-fa8cd091a4f0b3d9d0cb-7c7808508bf660d515c2719ea5 comes from the text below which describes the true lattice extreme lattice point coordinates and corresponding volume:

The 3 dimensional integer lattice spans between diagonal points (0, 0, 0) to (36, 46, 72) and volume 119,232.

This puzzle was inspired by a famous real-world problem, "The German Tank Problem," from World War II. There is a thorough Wikipedia article on this problem. What is amazing about the historical story is that simple statistical methods applied to data on the serial numbers of captured German tanks produced dramatically more accurate estimates of German tank production than more elaborate conventional military intelligence analysis.

This puzzle is essentially a three-dimensional version of the German Tank Problem. One slight difference is that the lattice coordinates have a minimum value of 0, whereas serial numbers have a minimum value of 1. This can be handled by adding 1 to each of the sample coordinates and then subtracting 1 from the estimated values in the formulas used.

Figure 1.

Max Lattice Coordinate	Sample Size	Largest Observed Value Plus 1	Point Estimate Formula Minus 1	Variance of Point Estimate Formula	2nd Moment of Point Estimate Formula
L	10	32	33.2	6.61200	1108.85200
M	10	47	49.7	16.77325	2486.86325
N	10	70	75.0	41.16667	5666.16667

Here is one estimation approach. Two key frequentist estimation formulas are:

$\hat{N}= m(1+1/k)-1$

where $\hat{N}$ is the estimated maximum serial number, m is the largest observed serial number, and k is the sample size, and

$Var(\hat{N})=((N-k)(N+1))/(k(k+2))$.

Figure 1 is a table of calculations for the three dimensions of the lattice.

So, the point volume estimate for the lattice is naturally $\hat{L}\hat{M}\hat{N}= 33.2 \times 49.7 \times 75=123{,}753$ and the variance of this volume estimator is:

$Var(\hat{L}\hat{M}\hat{N})=E[(\hat{L}\hat{M}\hat{N})^2]-E[\hat{L}\hat{M}\hat{N}]^2$
$=E[\hat{L}^2]E[\hat{M}^2]E[\hat{N}^2]-E[\hat{L}]^2E[\hat{M}]^2E[\hat{N}]^2$

which is naturally estimated as:

$=1108.85200 \times 2486.86325 \times 5666.16667-(123{,}753)^2$

$= 310{,}008{,}177.460093$

$StDev(\hat{L}\hat{M}\hat{N})=\sqrt{310{,}008{,}177.460093}=17{,}607.04908$.

John Berglund, Bob Conger, Glenn Meyers and Rob Thomas also submitted solutions.

Know the answer? Send your solution to ar@casact.org.