EXPLORATIONS BY GLENN MEYERS

A Cost of Capital Risk Margin Formula for Loss Reserve Liabilities

Readers of my previous columns (and other works) will know that I have been devoting a lot of effort on predicting the distribution of outcomes for stochastic loss reserve models. From the very beginning, the question that has always been in the back of my mind is "Why do this?"

I have never believed that the actuary's role is to derive a range for a loss reserve, and then support someone else's decision to post a reserve that is somewhere within that range. Instead, I believed that reserve estimates should have some kind of a risk margin.

While the subject was not loss reserving, one can see that line of thought in my 1991 CAS *Proceedings* paper, "The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking." If one looks at the *PCAS* articles that were written around that time, one will find that risk loading was a very hot topic. There was one article that made a lasting impression on me: the 1990 paper by Rodney Kreps, "Reinsurer Risk Loads from Marginal Surplus Requirements."

While I felt good when I was able to establish that my risk load formula could be viewed as marginal capital formula; I eventually realized that holding capital over time also had a cost. Therefore a "Risk Load as the Marginal Cost of Capital" idea needed refining to take into account how long an insurer needed to hold capital to support that risk. And that problem is relevant when determining a risk margin for loss reserves.

About two decades later, the European Union was in a position to recommend a liability risk margin formula called the Solvency II risk margin, which had many properties of a true cost of capital risk margin formula.

Let's now describe the cost of capital risk margin approach taken by this article.

Initially, an insurer will take a loss triangle and fit a stochastic loss reserve model to calculate its "best estimate" (defined as the present value of its unpaid loss liability) and the amount of capital needed to support that liability. At the end of the next year, more data will come in and the insurer will update its best estimate and the amount of capital needed to support that liability. Since we expect the best estimate to be more accurate, we expect that the capital needed to support that liability to be reduced, with the excess capital being returned to the insurer's investors. As this process continues, the insurer expects to receive a series of excess capital payments. A cost of capital risk margin reflects the insurer's cost of using their investors' capital.

To model this, let $t = 0, 1, \ldots$ be the time in years from the beginning of the original reporting date of a loss triangle. The examples in this article will be Schedule P loss triangles. Let T_t be a loss trapezoid where T_0 is the original loss triangle, and T_t is the loss trapezoid consisting of the original loss triangle plus data reported through the first t calendar years.

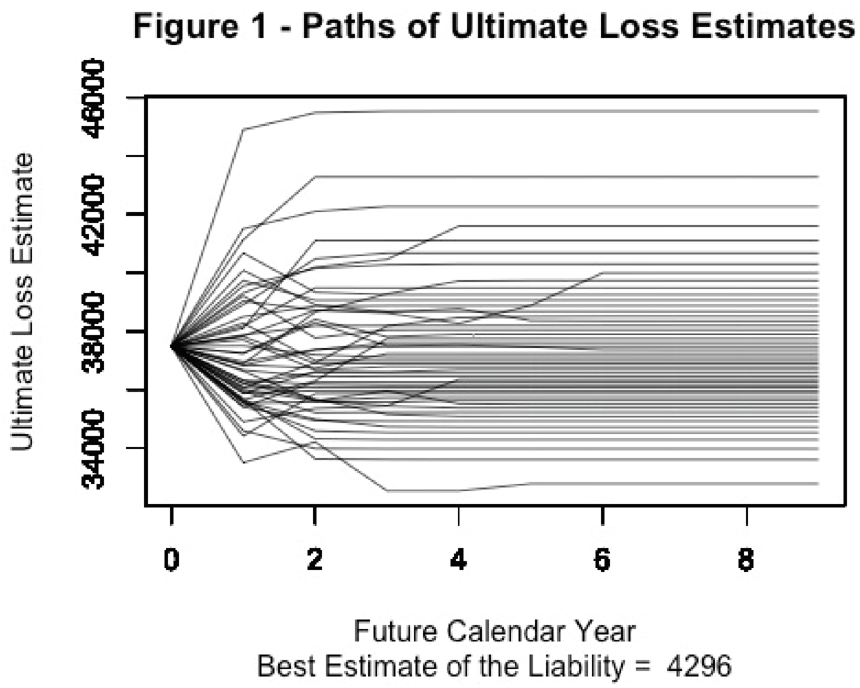
Given T_t, let E_t be the estimate of the expected ultimate loss reported at the end of calendar year t and let A_t be the amount of assets needed to support the uncertainty of E_t. A portion of A_t is supplied by policyholder premiums which we take to be equal to E_t. The remaining portion of A_t, $C_t \equiv A_t - E_t$, must be supplied by the insurance company's investors. C_t is called the insurer's required capital at the end of time t.

Let's assume that the insurer maintains C_t, at each time t, in a fund that earns a risk-free interest rate, i. To compensate for the risk of losing some (or all) of their capital, the insurer's investors will demand a higher return, $r > i$, on their investment, C_t. Let's look at the investor's cash flow.

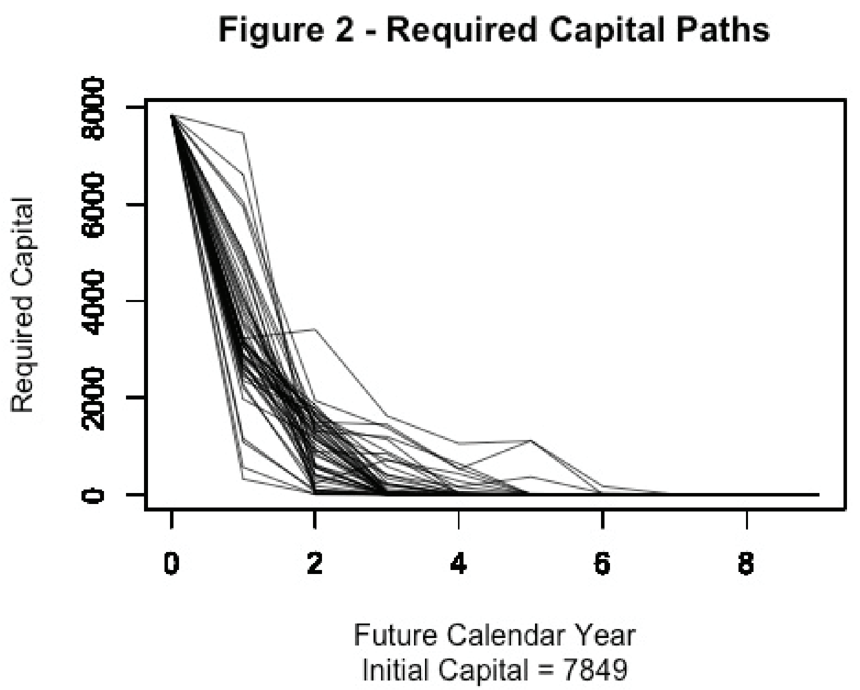
- At time $t = 0$, the insurer uses the information, T_0, to calculate the required initial capital investment, C_0.
- At time $t = 1$, the insurer uses the information, T_1, to calculate the required capital investment, C_1. It returns $C_0 \cdot (1+i) - C_1$ to the investor.[1]
- …
- At time t, the investor uses the information, T_t, to calculate the required capital investment, C_t. It returns $C_{t-1} \cdot (1+i) - C_t$ to the investor.
- …

The present value, discounted at the risky rate, r, of the

[1] Depending on T_t, the amount returned could be negative, resulting to an addition to the insurer's capital.



This chart illustrates the general tendency for the ultimate loss estimate to spread out over time.



This chart illustrates the general tendency for the required capital to decrease to zero over time.

amount returned, is equal to $\sum_{t=1}^{\infty} \frac{C_{t-1}\cdot(1+i) - C_t}{(1+r)^t}$. Since $r > i$, this present value is usually less than the initial capital investment of C_0. To adequately compensate the investor for taking on the risk of insuring policyholder losses, the difference can be made up at time $t = 0$ by what we now define as the cost of capital risk margin, R_{COC}.

$$\text{Cost of Capital Risk Margin} \equiv R_{COC} \equiv C_0 - \sum_{t=1}^{\infty} \frac{C_{t-1}\cdot(1+i) - C_t}{(1+r)^t}$$

$$= (r - i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1+r)^t}$$

with the last equality coming after some algebraic manipulations.

Note that R_{COC} is similar to, but not identical to, the Solvency II risk margin.

$$R_{SII} \equiv (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+i)^t}.$$

The issue that remains is how do we get the C_ts? To do this we make the assumption that we can use the output from a Bayesian MCMC stochastic loss reserve model to represent the set of future loss developments. As an example, let's use the CSR[2] model to generate 10,000 equally likely lognormal parameter sets $\{\mu^j_{wd}, \sigma^j_d\}_{j=1}^{10,000}$ for accident years $w = 1,\ldots,10$ and development years $d = 1,\ldots,10$ of Insurer 353 for the commercial auto line in the CAS Loss Reserve Database. Let's initially assume that we know which parameter set, $\{\mu^j_{wd}, \sigma^j_d\}$, we have. Then the estimate of the expected loss is the total expected loss over all accident years for the latest development period, 10 is given by

$$E^j_t = \sum_{w=1}^{10} e^{\mu^j_{w10} + (\sigma^j_{10})^2/2} \text{ for } t = 0,\ldots,9.$$

Given j, there is no uncertainty in the loss estimate, E^j_t, so $C^j_t = 0$.

Let's now drop the assumption that we know j.

So given T_t, there is uncertainty as to which parameter set, $\{\mu^j_{wd}, \sigma^j_d\}$ generated the losses. Since each parameter set is equally likely, $\Pr[\{\mu^j_{wd}, \sigma^j_d\}|T_t]$ is proportional to the likelihood of T_t given $\{\mu^j_{wd}, \sigma^j_d\}$.

Given these conditional probabilities, there are many ways to calculate E_t and A_t. I chose to take a sample S, with replacement, of size 10,000 from $\{E^j_t\}_{j=1}^{10,000}$ with sampling probabilities $\Pr[\{\mu^j_{wd}, \sigma^j_d\}|T_t]$. We set E_t equal to the mean of S, and A_t equal to the mean of the largest 300 elements of S.[3]

For a given $\{T_t\}_{t=0}^{9}$, the paths of E_t and C_t for $t = 0,\ldots,9$ can be plotted. Figures 1 and 2 show plots of these paths for several randomly selected $\{T_t\}_{t=0}^{9}$s from the model.[4] These plots illustrate the general tendency for the $\{E_t\}$ paths to spread out over time, and for the $\{C_t\}$ paths to decrease toward zero over time. One can then calculate a cost of capital risk margin by the above formulas, with $i = 4\%$ and $r = 10\%$, for each of 10,000 randomly selected $\{T_t\}_{t=0}^{9}$s. The average risk margin for our example was 717.

Under Solvency II, the risk margins for each line are added together, with no recognition of diversification for multiple lines. In my Explorations column for May/June *Actuarial Review*,[5] I argued that the independence assumption for the CSR model was appropriate. In taking the sample $S = S_1$ for commercial auto described above, and a similarly constructed sample S_2 for personal auto, under the independence assumption, I defined $S = S_1 + S_2$ to produce a combined risk margin with S being used exactly as I described above for a single line. The average risk margin for personal auto was 744. The sum of the risk margins in the two lines is 1,461 while the sum under the independence assumption is 1,025, indicating that a sizeable diversification benefit is appropriate for this example.

So, given that there is a sizeable diversification benefit, it would seem appropriate to assign a risk margin for a line that is proportional to the marginal contribution of each line to the insurer's total risk margin. ●

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[2] I used the version of the CSR model that is in my paper on dependencies published in the 2016 Winter *E-Forum*.

[3] This selection sets the C_0 approximately equal to 99.5% Value-at-Risk for the one-year time horizon that is specified by Solvency II.

[4] The "best estimate" of the liability is the average present value of the unpaid loss discounted at the risk-free interest rate, *i*.

[5] http://bit.ly/ARExplorMJ16